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EXPLICIT GUIDANCE OF A ROCKET WITH NON-UNIFORM THRUST†

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Abstract—Most of the available closed-loop explicit guidance algorithms either use constant thrust or constant acceleration thrust models. These assumptions which simplify the computation of thrust integrals are too restrictive for non-uniform thrusts. The inaccuracy in the thrust modelling works equivalent to corresponding perturbations in thrust values, and manifests in large changes in steering angles as well as inaccuracy in the injection condition and non-optimality. In this paper, a non-uniform thrust is modelled as linearly varying either increasing, decreasing or of constant magnitude over limited time segments. The corresponding thrust integrals are developed in a simple way. An earlier developed algorithm is extended where each of these linearly varying thrust segment is considered equivalent to a stage of a multi-stage rocket while solving the guidance problem. These thrust integrals can be used by most of the explicit guidance algorithms and help in improving performance considerably with non-uniform thrust. The numerical simulation results show that the algorithms using these thrust integrals provide significantly better results as compared to ones using constant thrust models or a model using remaining thrust constant.

1. INTRODUCTION

In the existing explicit guidance algorithms[1–4] for launch vehicles, thrust is assumed to be constant or throttled to produce constant acceleration. Often these assumptions are too restrictive. In many cases, the thrust varies significantly with time, particularly in the case of solid fuel rockets. In order to solve for the optimum steering problem of closed-loop guidance involving varying thrust, the scheme for constant thrust case can still be used if thrust integrals for varying thrust of the form (F, thrust; m, mass; T_{go}, time-to-go; t, time)

$$\int_0^{T_{go}} \{F(t)t^n/m(t)\}\,dt$$

and

$$\int_0^{T_{go}} \int_0^{t} \{F(t)t^n/m(t)\}\,dt\,dt$$

are available. The estimates of these thrust integrals are often made based on the average for the stage and corresponding mass flow-rate. In a recent study, Shrivastava *et al.*[5] used the remaining mass up to the end of stage to compute the moving average mass flow-rate for the computation of thrust integrals. The results corresponding to these two approaches are unsatisfactory in accuracy and stability. A varying thrust, unless modelled accurately, produces effects equivalent to perturbations in thrust and mass and hence puts a greater demand on guidance. Consequently this equivalent perturbation, which is felt more when the remaining time-to-go is small, results in higher injection inaccuracies and instabilities (seen as higher turning rate of thrust vector). When only an intermediate stage has a varying thrust, but not the final stage, the effect is not so critical as compared to the case where the varying thrust is in the final stage. This necessitates a more accurate estimate of effects of thrust.

2. ANALYSIS

Any predefined varying thrust can be idealized as linearly varying for a limited part of thrust–time profile. This way, the whole profile can be divided into a number of segments over which the thrust can be assumed to vary linearly, (Fig. 1). The number of segments depends upon the nature of thrust variation and accuracy desired.

Let the vacuum thrust in the jth segment, F_j be given by

$$F_j(t) = V_{ex}\dot{m}_j(t) \tag{1}$$

where V_{ex} is constant effective exhaust velocity for the entire stage comprising a number of segments, so that thrust during jth segment, F_j is directly proportional to the mass flow-rate, $\dot{m}_j$. Let the absolute value of the mass flow-rate for the jth segment be

$$\dot{m}_j = a_j t + b_j \quad 0 < t \leqslant T_j, \tag{2}$$

where a_j is the gradient of the mass flow-rate (or thrust) during the *jth* segment, b_j is the mass flow-rate

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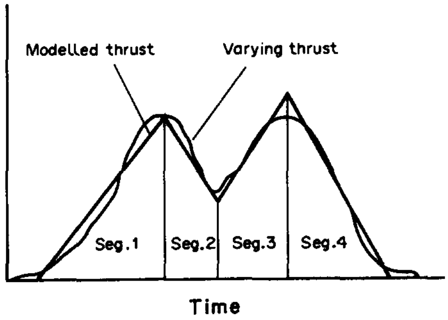


Fig. 1. Linear thrust model with four segments for a varying thrust stage.

at $t = 0$ during the jth segment, and T_j is the remaining burn time for jth segment, t is time within jth segment measured from the start of the segment (if the ignition of segment j has not started) or from the current time (if the current segment is jth segment). The mass at any time t during the jth segment is given by

$$m_j(t) = c_j - 1/2a_jt^2 - b_jt \tag{3}$$

where c_j is the mass of the vehicle at $t = 0$ during the jth segment. Since c_j and m_j represent mass,

$$c_j > 0,$$

and

$$m_j > 0.$$

The thrust in the segment j at anytime t is obtained by substituting eqn (2) into (1) and is given by

$$F_j(t) = V_{ex}(a_jt + b_j). \tag{4}$$

Evaluations of a_j, b_j, c_j are done *a priori* by making a linear least squares fit of the thrust over each segment. Since V_{ex} is known for a given motor, a_j and b_j can be computed from

$$F_j(t) = r_jt + s_j$$

where $r_jt + s_j$ is the line representing thrust in the least squares way in jth segment. Then, using eqn (4) one gets

$$a_j = r_j/V_{ex}, \tag{5a}$$

and

$$b_j = s_j/V_{ex}. \tag{5b}$$

Evaluation of thrust integrals

The thrust integrals for the jth segment are defined as follows:

$$I_{Sn_j} = \int_0^{T_j} \{F_j(t)t^n/m_j(t)\}\,dt \tag{6a}$$

$$I_{Dn_j} = \int_0^{T_j}\int_0^{t} \{F_j(t)t^n/m_j(t)\}\,dt\,dt, \tag{6b}$$

where T_j is the burn time for segment j.

As the mass flow-rate has been considered to be positive, $b_j > 0$. On the other hand, a_j may be positive, negative or zero in a segment depending on the nature of variation of thrust. When $a_j = 0$, it represents a constant thrust over the segment and the relations obtained for thrust integrals for constant thurst[1] are valid. For the cases of $a_j > 0$ and $a_j < 0$, an analytical solution for single integrals, I_{Sn_j}, are derived next. The double integrals, I_{Dn_j}, can be expressed in terms of I_{Sn_j} by carrying out integration, by parts, of eqn (6b) and using eqn (6a) as

$$I_{Dn_j} = T_jI_{Sn_j} - I_{S(n+1)_j}.$$

Let

$$x \triangleq b_j/a_j + t \tag{7a}$$

and

$$l^2 \triangleq b_j^2/a_j^2 + 2c_j/a_j. \tag{7b}$$

Case 1. Linearly increasing thrust $(a_j > 0)$

On substituting expressions for F_j, m_j in expression for I_{Sn_j}, given by eqn (6a), one gets

$$I_{Sn_j} = V_{ex}\int_0^{T_j} t^n(a_jt + b_j)/(c_j - 1/2a_jt^2 - b_jt)\,dt. \tag{8}$$

Rewriting:

$$t^n/(c_j - 1/2a_jt^2 - b_jt) = -2t^{n-2}/a_j + \{2(c_j/a_j)t^{n-2} - 2(b_j/a_j)t^{n-1}\}/(c_j - 1/2a_jt^2 - b_jt).$$

On substituting the above relation in eqn (8) one obtains

$$I_{Sn_j} = V_{ex}\left[-\frac{2t^n}{n} + \frac{2b_jt^{n-1}}{a_j(n-1)}\right]_0^{T_j} + 2(b_j/a_j)I_{S(n-1)_j} - 2(c_j/a_j)I_{S(n-2)_j}. \tag{9}$$

I_{Sn_j} can be solved recursively for $n \geqslant 2$ if I_{S0_j} and I_{S1_j} are known. Equation (8) can be rewritten as

$$I_{Sn_j} = -V_{ex}\int_0^{T_j} t^n\,d/dt\{\ln(c_j - 1/2a_jt^2 - b_jt)\}\,dt. \tag{10}$$

For $a_j > 0$, $l^2 > 0$ as $c_j > 0$. So, l is a real number which is taken to be positive square-root of $(b_j^2/a_j^2 + 2c_j/a_j)$. On using eqns (7a) and (7b), one obtains

$$m_j(t) = c_j - 1/2a_jt^2 - b_jt = a_j(l^2 - x^2)/2 > 0 \tag{11}$$

which leads to $(l - x) > 0$.

From eqn (7a) one gets

$$t^n = x^n[1 - b_j/(a_jx)]^n = x^n\left[1 + \sum_{r=1}^{n} {}^nC_r(-1)^r(b_j/a_jx)^r\right] \tag{12}$$

where

$${}^nC_r = n!/\{(n-r)!r!\}.$$

On substituting eqns (11) and (12) into eqn (10) and expanding the terms one gets

$$-I_{Sn_j}/V_{ex} = \int_0^{T_j} x^n \, d/dt\{\ln(l^2 - x^2)\}\,dt$$

$$-nb_j/a_j \int_0^{T_j} x^{n-1}\, d/dt\{\ln(l^2 - x^2)\}\,dt$$

$$+\cdots+(-1)^n b_j^n/a_j^n \int_0^{T_j} d/dt\{\ln(l^2 - x^2)\}\,dt. \quad (13)$$

Let

$$I'_{1Sn_j} \triangleq \int_0^{T_j} x^n \, d/dt\{\ln(l^2 - x^2)\}\,dt.$$

Equation (13) can be rewritten as

$$I_{Sn_j} = -V_{ex}[I'_{1Sn_j} - nb_j/a_j I'_{1S(n-1)_j} + \cdots + (-1)^n (b_j/a_j)^n I'_{1S0_j}]. \quad (14)$$

Equation (14) can be evaluated for I_{S0_j} and I_{S1_j} by substituting $n = 0$ and 1, respectively, as follows

$$I_{S0_j} = -V_{ex} I'_{1S0_j}$$

$$I_{S1_j} = -V_{ex}(I'_{1S1_j} - b_j I'_{1S0_j}/a_j).$$

After integrating by parts for integral I'_{1Sn_j} one obtains

$$I'_{Sn_j} = [x^n \ln(l^2 - x^2)]_0^{T_j} - n\int_0^{T_j} x^{n-1} \ln(l^2 - x^2)\,dt. \quad (15)$$

On using eqn (15), one can evaluate I'_{1Sn_j} for $n = 0$ and 1 as follows

$$I'_{1S0_j} = [\ln(l^2 - x^2)]_0^{T_j}$$

$$I'_{1S1_j} = [x \ln(l^2 - x^2)]_0^{T_j} - [x\ln(l^2 - x^2) - 2x + l\ln\{(l + x)/(l - x)\}]_0^{T_j}$$

where x and l are defined by eqn (7).

Case 2. Linearly decreasing thrust ($a_j < 0$) *and* $l^2 > 0$

Since $l^2 > 0$, l is a real number and is taken as positive square root of l^2. $m_j(t)$ can be written as

$$m_j(t) = (-1/2a_j)(x^2 - l^2), \quad (16)$$

for $a_j < 0$, $(x^2 - l^2) > 0$.

So, $(x - l)$ and $(x + l)$ are both of the same sign, either positive or negative, and hence $\ln\{(x - l)/(x + l)\}$ is always defined.

On using eqn (16), a relation similar to eqn (13) can be written as

$$-I_{Sn_j}/V_{ex} = \int_0^{T_j} x^n\left[1 + \sum_{r=1}^{n} n_{C_r}(-1)^r\{b_j/a_j x\}^r\right] d/dt(x^2 - l^2)\,dt. \quad (17)$$

Let

$$I'_{2Sn_j} \triangleq \int_0^{T_j} x^n \, d/dt\{\ln(x^2 - l^2)\}\,dt$$

where integrals I'_{2Sn_j} correspond to Case 2. Equation (17) can be rewritten as

$$I_{Sn_j} = -V_{ex}[I'_{2Sn_j} - nb_j/a_j I'_{2S(n-1)_j} + \cdots + (-1)^n (b_j/a_j) I'_{2S0_j}]. \quad (18)$$

I_{Sn_j} for $n \leqslant 1$ is obtained from eqn (17) as follows:

$$I_{S0_j} = -V_{ex} I'_{2S0_j}$$

$$I_{S1_j} = -V_{ex}[I'_{2S1_j} - b_j I'_{2S0_j}/a_j]_0^{T_j}.$$

On integrating the expression for I'_{2Sn_j} by parts, one gets

$$I'_{2Sn_j} = [x^n \ln(x^2 - l^2)]_0^{T_j} - n\int_0^{T_j} \ln|(l^2 - x^2)|x^{n-1}\,dt. \quad (19)$$

Using eqn (19) one finds

$$I'_{2S0_j} = [x\ln(x^2 - l^2)]_0^{T_j}$$

$$I'_{2S1_j} = [x\ln(x^2 - l^2)]_0^{T_j} - \left[x\ln|(l^2 - x^2)| - 2x + n\left|\frac{l + x}{l - x}\right|\right]_0^{T_j}$$

where x and l are defined by eqn (7).

Case 3. $a_j < 0$ *and* $l^2 < 0$

Let

$$l'^2 \triangleq -(b_j^2/a_j^2 + 2c_j/a_j) \quad (20)$$

then

$$l'^2 > 0 \text{ for } (b_j^2 + 2c_j a_j) < 0$$

so

$$m_j(t) = c_j - 1/2a_j t^2 - b_j t = (-1/2a_j)(x^2 + l'^2).$$

On using eqn (20), a relation similar to eqn (13) can be written as

$$-I_{Sn_j}/V_{ex} = \int_0^{T_j} x^n\left[1 + \sum_{r=1}^{n} n_{C_r}(-1)^r(b_j/a_j x)^r\right] d/dt\{\ln(x^2 - l'^2)\}\,dt. \quad (21)$$

Let

$$I'_{3Sn_j} \triangleq \int_0^{T_j} x^n \, d/dt\{\ln(x^2 + l'^2)\}\,dt$$

where I'_{3S_n} corresponds to Case 3.

Then

$$I_{Sn_j} = -V_{ex}[I'_{3Sn_j} - nb_j/a_j I'_{3S(n-1)_j} + \cdots + (-1)^n (b_j/a_j)^n I'_{3S0_j}]. \quad (22)$$

I_{Sn_j} can be solved for $n \geqslant 2$ if I_{S0_j}, I_{S1_j} are known. After substituting $n = 0$ and 1 in eqn (22), one gets I_{Sn_j} for $n \leqslant 1$ as follows

$$I_{S0_j} = -V_{ex} I'_{3S0_j}$$

$$I_{S1_j} = -V_{ex}[I'_{3S1_j} - b_j/a_j I'_{3S0_j}].$$

Integration by parts of the expression for I'_{3Sn_j} yields,

$$I'_{3Sn_j} = [x^n \ln(x^2 + l^2)]_0^{T_j} - \int_0^{T_j} nx^{n-1} \ln(x^2 + l'^2)\,dt. \quad (23)$$

The integrals given by eqn (23) can be evaluated for $n = 0$ and 1 as follows:

$$I'_{3SO_j} = [n(x^2 - l'^2)]_0^{T_j}$$

$$I'_{3S1_j} = [x \ln(x^2 + l'^2)]_0^{T_j} - [x \ln(x^2 + l^2)$$

$- 2x + 2l' \tan^{-1}(x/l')]_0^{T_j}$ where x is defined by eqn (7) and l' is defined by eqn (20).

3. SOLUTION OF THE GUIDANCE PROBLEM USING VARIABLE THRUST

Having obtained the thrust integrals for each segment the system can be treated as a multi-stage rocket where each segment works like an equivalent single stage. The solution where guidance is operative in more than one stage is treated in detail in [1]. Using the method developed, changes in position and velocity can be determined with the number of segments in the varying thrust stage taken to be equivalent to number of stages.

Using the algorithm for the solution of guidance problem given in [1], effects of thrust in terms of changes in velocity and position corresponding to each of the segment for a given set of guidance parameters can be determined. The corresponding changes due to gravity can also be estimated using Encke's method[1].

Effects of thrust

The required change in velocity to be inparted by thrust is given by

$$\mathbf{v}_{thR} \triangleq \mathbf{v}_T - \mathbf{v}_0 - \mathbf{v}_g$$

where $\mathbf{v}_T$ is the required velocity at injection, $\mathbf{v}_o$ is the current velocity and

$$\mathbf{v}_g \triangleq \int_0^{T_{go}} \mathbf{g}\{\mathbf{r}(t)\}\,dt.$$

$\mathbf{v}_g$ can be estimated using Encke's method[1]. (**Bold** letters indicate vectors.)

The optimal unit thrust vector, developed in [1] is given in terms of unit vectors $\hat{\mathbf{I}}_{xc}$ and $\hat{\mathbf{I}}_{yc}$ by

$$\mathbf{u}(t) = [\hat{\mathbf{I}}_{xc} + \hat{\mathbf{I}}_{yc} p_{\dot\theta}(t - \tau_c)]/[1 + p_{\dot\theta}^2(t - \tau_c)^2]^{1/2} \quad (24)$$

where $\hat{\mathbf{I}}_{xc}$ and $\hat{\mathbf{I}}_{yc}$ form the plane of correction, $p_{\dot\theta}$ is the tangent of the steering angle made from $\hat{\mathbf{I}}_{xc}$, and τ_c is a time constant. The denominator of eqn (24) can be written as

$$[1 + p_{\dot\theta}^2(t - \tau_c)^2]^{1/2} = 1 - 1/2p_{\dot\theta}^2(t - \tau_c)^2 + 3/8p_{\dot\theta}^4(t - \tau_c)^4 + \cdots. \quad (25)$$

The estimated change in velocity imparted by thrust for a specified set of guidance parameters for a $p_{\dot\theta}$-segment varying thrust stage is given by

$$\mathbf{v}_{thA} \triangleq \hat{\mathbf{I}}_{xc} \sum_{j=\mathrm{c.s.}}^{p} \int_0^{T_j} F_j/[m_j\{1 + p_{\dot\theta}^2(t - \tau_{c_j})^2\}^{1/2}]\,dt$$

where c.s. is current segment, $\hat{\mathbf{I}}_{xc}$ is the direction in which velocity constraint is met, and T_j is the remaining burn time for the jth segment. τ_{c_j} for the current segment can be computed using the following relation

$$0 = \sum_{j=\mathrm{c.s.}}^{p} \int_0^{T_j} F_j p_{\dot\theta}(t - \tau_{c_j})/[m_j\{1 + p_{\dot\theta}^2(t - \tau_{c_j})^2\}^{1/2}]\,dt.$$

τ_{c_j} of any other segment can be computed with respect to τ_{c_j} of the current segment as follows

$$\tau_{c_{j+n}} = \tau_{c_j} - \Delta t_{j+n},$$

where τ_{c_j} is for the current segment and Δt_{j+n} is the time-difference between ignition time of $(j+n)$ segment and the current time.

The expression for $\mathbf{r}_{thA}$ can also be written in a similar manner on using the following relation

$$\begin{aligned}\mathbf{r}_{thA} = \hat{\mathbf{I}}_{xc} \sum_{j=\mathrm{c.s.}}^{p} \Bigg[&\int_0^{T_j} \int_0^{t} F_j(s)/[m_j(s) \\ &\times \{1 + p_{\dot\theta}^2(t - \tau_{c_j}\}^{1/2}]\,ds\,dt \\ &+ t_{r_j} \int_0^{T_j} F_j(t)/[m_j(t)\{1 + p_{\dot\theta}^2(t - \tau_{c_j})^2\}^{1/2}]\,dt\Bigg] \\ + \hat{\mathbf{I}}_{yc} \sum_{j=\mathrm{c.s.}}^{p} \Bigg[&\int_0^{T_j} \int_0^{t} F_j(s) p_{\dot\theta}(s - \tau_c)/[m_j(s) \\ &\times \{1 + p_{\dot\theta}^2(s - \tau_{c_j})^2\}^{1/2}]\,ds\,dt \\ &+ t_{r_j} \int_0^{T_j} F_j(t)/[m_j(t)\{1 + p_{\dot\theta}^2(t - \tau_{c_j})^2\}^{1/2}]\,dt\Bigg]\end{aligned}$$

where t_{r_j} is the remaining time for the thrust cut-off after the end of jth segment. The corresponding required change in position to meet the end constraints can be given as

$$\mathbf{r}_{thR} = \mathbf{r}_T - \mathbf{r}_o - \mathbf{v}_o T_{go} - \mathbf{r}_g,$$

where $\mathbf{r}_T$ is the position at injection, $\mathbf{r}_o$ is the current position, and $\mathbf{r}_g$ is defined as

$$\mathbf{r}_g \triangleq \int_0^{T_{go}} \int_0^{t} \mathbf{g}\{\mathbf{r}(s)\}\,ds\,dt.$$

$\mathbf{r}_g$ can be estimated using Encke's method[1].

For the solution, the following constraints must be satisfied

$$\mathbf{v}_{thR} = \mathbf{v}_{thA},$$

and

$$\mathbf{r}_{thR} = \mathbf{r}_{thA}.$$

Using eqn (25), the expression for $\mathbf{v}_{thA}$ and $\mathbf{r}_{thA}$ can be written in terms of I_{sn_j} and I_{Dn_j} for each segment and thus $\mathbf{v}_{thA}$ and $\mathbf{r}_{thA}$ can be evaluated.

The steering parameters used in eqn (24) can be determined after forming algebraic constraint equations derived from vector constraint equation given

Table 1. Variable thrust–time and mass profile

Time from guidance initiation (s)	Mass (kg)	Thrust (N)
0	4300.00	11250
25	4201.04	12500
50	4092.71	13500
75	3975.63	14600
100	3846.67	15650
125	3712.71	16500
150	3572.09	17250
175	3425.52	18000
200	3282.50	18600
225	3125.83	19000
250	2966.67	19000
275	2812.51	18000
300	2668.76	16500
325	2538.34	14800
350	2420.00	13400
365	2367.67	9800
375	2330.75	8300
400	2300.00	0

above. These constraint equations are solved using differential correction method[1] where partial derivatives are found analytically.

4. NUMERICAL SIMULATIONS AND RESULTS

In order to investigate the effectiveness of the algorithm with varying thrust, numerical simulations are conducted. For this, the performance of the guidance in the final stage with varying thrust and having thrust cut-off facility is considered.

The variation of mass of the vehicle and thrust-time profile of this stage are given in Table 1. Figure 2 shows the variation of the thrust with time considered for simulation. Table 2 gives the initial position and velocity of the rocket at the start of guidance. The desired orbit at injection is also given in Table 2, which corresponds to a 900 km circular Sun-synchronous orbit.

Table 3(a) shows the idealization of the varying thrust given in Table 1 by different models. Three

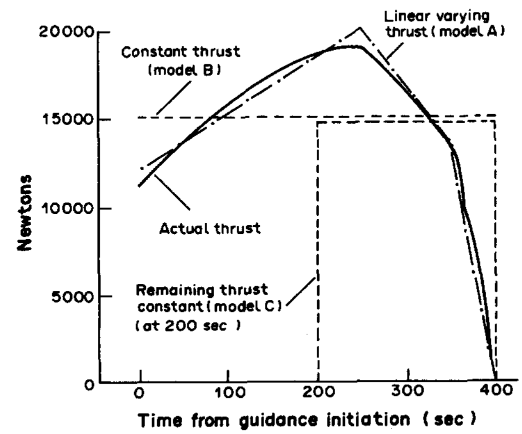


Fig. 2. Thrust–time profiles using a linear time-varying thrust model, constant thrust model and moving average constant thrust model.

Table 2. Initial and target conditions of the vehicle

Initial Position (x,y,z components)	(2181127.34, −1978553.22, 6570848.69) m
Initial velocity (x,y,z components)	(3519.09498, −4212.32, −1766.37778) m/s
Geocentric distance at injection	7278137.0 m
Inertial velocity at injection	7440.46107 m/s
Flight path angle at injection	0.0°
Inclination at injection	98.98°
Longitude of the ascending node (measured from the Greenwich meridian at lift off)	263.76°

such models are considered. The first model (model A) corresponds to the linearly varying thrust model developed in this paper, where the thrust–time profile is divided into three segments. The first segment is from the start of the state up to 250 s where thrust is assumed to vary linearly from 12,000 to 20,000 N. In the next segment, from 250 to 350 s, the thrust is assumed to vary from 20,000 N to 13,500 N linearly. In the last segment, from 350 to 400 s, the thrust is assumed to drop from 13,500 N to zero linearly. The corresponding values of coefficients a,b,c for these three segments are shown in Table 3(b).

The model is investigated for both increasing and decreasing thrust. The other two models considered for comparing its performance are similar to those discussed by Shrivastava *et al.*[5]. Model B corresponds to a constant thrust of 15,000 N for the entire stage which has nearly the same total impulse as the varying thrust given in Table 1; and model C corresponds to a constant mass flow-rate obtained by taking the ratio of remaining propellant in the stage and the maximum possible duration of burn of the stage. For this duration the thrust, which is computed corresponding to the moving average mass flow-rate, is assumed to be constant. Figure 2 also shows the thrust–time profile assumed by model B and the constant thrust considered by Model C at 200 s after the ignition of the stage which has a maximum duration of burn equal to 400 s.

For investigating the performance of these models, the guidance algorithm described in [1] is used. The first case corresponds to an ideal situation where the

Table 3. (a) Assumed thrust–time profiles by models A,B,C

Time from guidance initiation (s)	Thrust using model A (N)	Thrust using model B (N)	Thrust using model C (N)
0	12000	15000	15000
200	18400	15000	14737.5
250	20000	15000	13333.2
350	13500	15000	7200
400	0	0	0

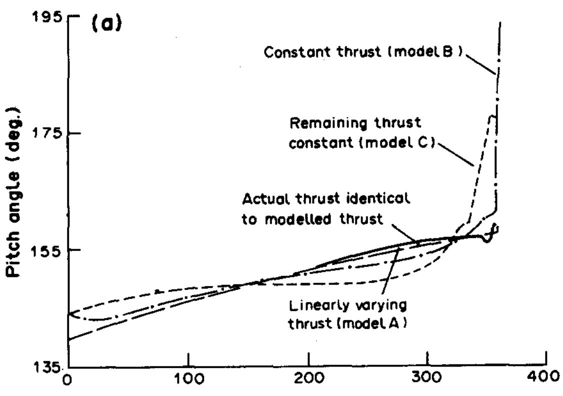
Table 3. (b) Coefficients a,b,c for three segments for Model A

Time from guidance initiation (s)	Segment No. (j)	Coefficient a_j (kg/s^2)	Coefficient b_j (kg/s)	Coefficient c_j (kg)
0	1	0.01066667	4.0	4300
250	2	−0.023333	6.66667	1966.667
350	3	−0.093333	4.66667	2416.667

Table 4. Injection errors using different models

	Ideal	Model A	Model B	Model C
Burn time (s)	366.21	366.03	366.23	366.954
Error in semi-major axis (m)	−0.009	−2.12	16561.9	497.6
Error in eccentricity	2.62×10^{-8}	6.0×10^{-5}	2.52×10^{-3}	1.10×10^{-3}
Error in injection position (x,y,z component) (m)	(0.075, −0.12, −0.13)	(177.22, −274.50, 291.90)	(−4497.41, 7081.09, 7529.4)	(3214.63, −5084.77, −5422.06)
Error in injection velocity (x,y,z components) (m)	$(-0.11, 0.16, -9.1) \times 10^{-6}$	$(-0.20, 0.82, 1.04) \times 10^{-3}$	(1.53, −9.69, −2.63)	(−0.07, 0.20, −0.21)

actual thrust is identical to the linear thrust of model B. The injection accuracy and burn time for this case are given in Table 4. The table also shows the injection accuracies and burn times when thrust corresponding to models A, B and C, respectively, are used. In the ideal case, the injection error is nearly zero. The errors when model A is used are 2.12 m in a semi-major axis and 6.0×10^{-5} in eccentricity which are quite small. As compared to these results, when models B and C are used, the errors in semi-major axis are 16561.9 and 467.6 m, respectively. The errors in eccentricity using models B and C are 2.522×10^{-3} and 1.103×10^{-3}, respectively. The pitch steering angles using the ideal case and models A, B and C are given in Fig. 3(a). Figure 3(b) shows the corresponding plots for yaw angles. It can be seen that the ideal case and model A follow almost linear variation of steering angles, whereas in case of B and C, the instability sets in after about 270 s of guidance and the situation worsens thereafter. To use models B and C without instability one has to remove position constraint nearly 100 s before the actual cut-off. This will introduce large injection errors. Thus, for a case where the final stage has a variable thrust, model A alone provides a satisfactory solution.



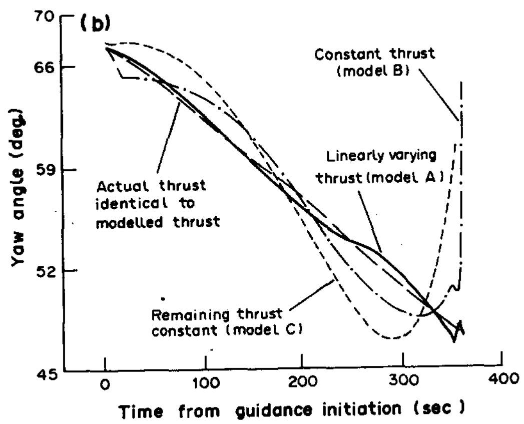


Fig. 3. Pitch (a) and yaw (b) angle profile using actual thrust identical to model thrust, linear time varying constant thrust and moving average constant thrust models.

5. CONCLUSION

In the published literature, the closed-loop guidance algorithm has been developed for constant acceleration and constant thrust for a rocket. In this paper an existing algorithm has been extended to include a varying thrust case. A model consisting of segments over which thrust varies linearly is used. The corresponding thrust integrals are developed for linearly increasing and decreasing thrust. This can be used along with most of the guidance algorithms developed earlier. The numerical simulation results show that the thrust integrals developed for varying thrust give significantly better results as compared to those using constant thrust model and the model using remaining thrust constant.

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